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Numerical evidence of ultrarefractive optics in photonic crystals

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Abstract

This paper is devoted to a numerical demonstration of a conjecture of specialists of photonic band structures: the existence of ultrarefractive optics phenomena. Near the edges of a transmission gap, the permittivity of a dielectric photonic crystal becomes close to zero. As a consequence, surprising refractive effects should be observed on the light transmitted and reflected by a slice of photonic crystal. A property of beam translation very similar to the Goos–Hanschen effect (but in transmission) is shown, as well as a strange phenomenon: a beam incident on a slice of photonic crystal can be enlarged or even split into some separate transmitted beams. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Arguments based on dispersion diagrams of light inside photonic crystals have allowed some specialists in this field to predict the phenomenon of ultrarefraction of light [1–4]. Comparable considerations can be found in Ref. [5] in the context of a modulated planar waveguide. In outline, light velocity at edges of a transmission gap may tend to infinity and thus a photonic crystal can simulate an effective medium having a permittivity close to zero.

In this paper, we have used numerical tools based on rigorous electromagnetic theories [6,7] in order to demonstrate the validity of these predictions. Some surprising applications of ultrarefractive optics will be shown. For example, field maps of transmitted field generated by a slice of photonic crystals illuminated by a light beam bear evidence of a phenomenon similar to the Goos–Hanschen effect [8]: the translation of the transmitted beam with respect to the location predicted by geometrical optics. In contrast to the classical Goos–Hanschen phenomenon the photonic crystal allows to get this translation phenomenon for the transmitted beam, and close to normal incidence. Even though the mathematical demonstration of the Goos–Hanschen effect does not hold in that context, it can be intuited that the beam translation phenomenon is linked to a rapid variation of the argument of the transmitted wave with incidence angle.

A heuristic reasoning allows one to predict a surprising phenomenon: the beam transmitted by a photonic crystal under ultrarefractive condition can be significantly widened with respect to the prediction of geometrical optics. Field maps show that this intuitive conjecture is right. Furthermore, it is shown that under some conditions, the transmitted light may be split into some separate beams.

2. The concept of effective medium

It can be noticed that the concept of effective medium near Brillouin zone edges has been used by Lin et al. [9] in

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order to realize a prism in a two-dimensional (2D) photonic crystal material which could serve as a dispersive element in a ultracompact miniature spectrometer. It has been shown in a recent paper [10] that a metallic photonic crystal can simulate a medium having an effective index close to zero. In this paper, we are concerned with dielectric photonic crystals, the properties of which are very different from those of metallic crystals. However, let us show that a simple reasoning can lead us to the same conclusions: a dielectric photonic crystal can simulate an effective medium having a permittivity close to zero.

Fig. 1 shows a slice of a 2D dielectric photonic crystal with square symmetry and Fig. 2 gives the transmission factor (in energy) and the phase shift of the transmitted wave with respect to the incident one, which is a s-polarized plane wave in normal incidence. It is important to notice that in the range of wavelengths $\lambda \in [2.4, 3.3]$, the only reflected and transmitted waves generated by this grating are the zero order ones. A transmission gap is obtained in the range $\lambda \in [2.5, 3.2]$.

Let us show that, at a given wavelength and a given incidence, this slice of photonic crystal can simulate a slice of homogeneous medium whose permittivity is close to zero. First, let us notice that the search for the parameters ε_h and e of the equivalent thin film defined in Fig. 3 is a well posed problem, at least if one is interested in the transmitted field only. Indeed, this transmitted field is characterized by the transmission coefficient t when the crystal is illuminated by a s-polarized wave with given incidence and wavelength. The equivalent thin film must give the same coefficient t when it is stroked by the same incident wave. In order to obtain this complex transmission coefficient, we can choose two parameters: the width e and the relative permittivity ε_h of the dielectric medium. Since the photonic crystal is a lossless structure, we will assume that the thin film is lossless as well, thus ε_h is real. In conclusion, the transmission coefficient t, which is a complex number, may be obtained by choosing two real numbers: e and ε_h. This reasoning shows that our search is consistent, but does not provide a rigorous proof of the existence of such a thin film. However, it exists a range of wavelengths where this existence can be predicted. Let us consider very large wavelengths (much larger than the size of the elementary cell of the photonic crystal). It is well known that, under these conditions, the heterogeneous structure can be homogenized. In this homogenization domain, the effective index of the crystal is given by simple rules [11]. For s-polarized light, the effective permittivity $\varepsilon_{h,\infty}$ of the medium is the mean value of ε in the elementary cell of the photonic crystal. Thus it is a real number greater than unity (right-hand side of Fig. 4). When the wavelength λ is reduced, we are inside the gap and the crystal becomes opaque: the light is exponentially attenuated during propagation. As a consequence, the permittivity ε_h must be a negative real number. Indeed, positive permittivities allow propagation, and complex permittivities imply losses and must also be rejected since the rods are lossless. The reader can notice that such a permittivity generates a pure imaginary optical index. If we conjecture that ε_h is a continuous function of λ, the curve must cross the abscissa axis, thus there exists a wavelength where $\varepsilon_h = 0$ (point B of Fig. 4). The same reasoning can be made for the left-hand side of Fig. 2. Indeed, for

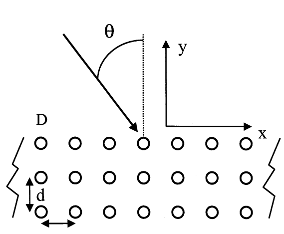

Fig. 1. Schema of a slice of a 2D dielectric photonic crystal. This crystal is made of seven grids of infinite extension in x with period $d = 1.27$, separated in y by the same distance d. Each grid is formed by parallel dielectric infinite rods of index $\nu = 3$ and diameters $D = 0.95$.

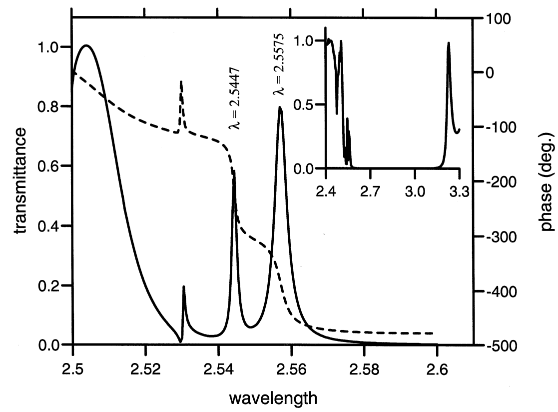

Fig. 2. Energy (solid line) and phase shift (dashed line) of the wave transmitted by the dielectric photonic crystal of Fig. 1 in normal incidence with s-polarized light versus the wavelength λ in vacuum. The small inserted graph shows the energy on a larger range of wavelengths.

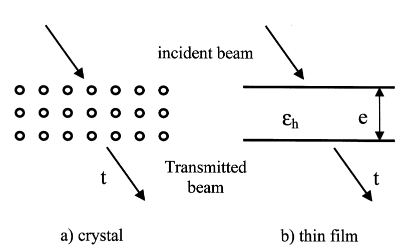

Fig. 3. Equivalence of a slice of photonic crystal (left) with a dielectric homogeneous thin film (right) of permittivity $\varepsilon_h = \nu_h^2$ (ν_h is the corresponding optical index).

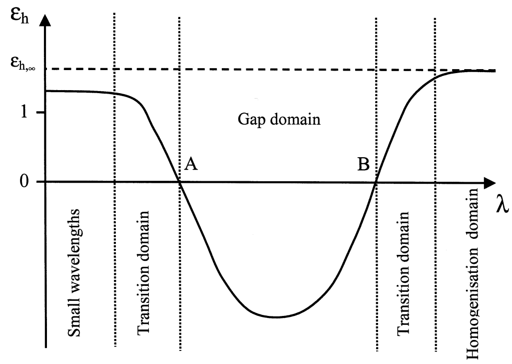

Fig. 4. Physical predictions of the variation of the effective permittivity ε_{h} of the thin film as a function of the wavelength under the conditions of Fig. 2.

smaller wavelengths the light can propagate again inside the crystal and thus the permittivity ε_{h} is positive, which shows that there exists a second wavelength for which the permittivity vanishes (point A of Fig. 4).

Of course, the value of ε_{h} for a given wavelength λ should depend on the angle of incidence θ. However, the goal of a photonic crystal is to provide light propagation properties which are almost independent of the direction of propagation.

3. Conjectures about ultrarefractive optics

Let us consider the dielectric thin film of Fig. 5, with a positive relative permittivity close to zero, illuminated by a monochromatic incident beam with incidence angle close to zero. Due to the refraction phenomenon, the beam inside the dielectric can propagate with a large refraction angle and the emerging beam below the thin film is shifted with respect to the incident beam. It is worth noticing that the direction of this shift is just opposite to that obtained for classical materials with permittivities larger than unity. Moreover, the amplitude of the shift can be very large if the permittivity is very close to zero. A slice of photonic

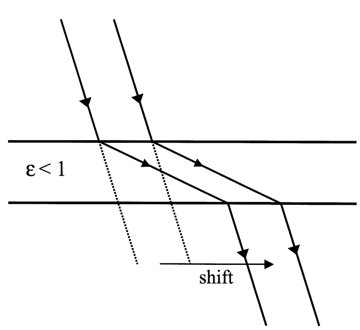

Fig. 5. Anomalous shift of the emerging beam after transmission by a thin film with small permittivity.

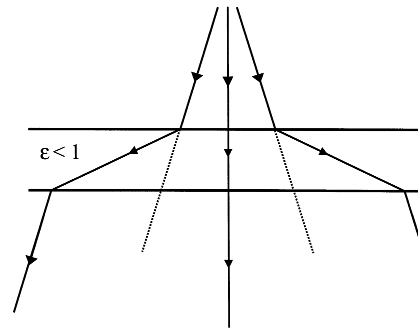

Fig. 6. Widening of the emerging beam after transmission by a thin film with small permittivity.

crystal illuminated by an incident beam at a wavelength close to the edge of a gap should generate the same phenomenon.

Now, let us illuminate the dielectric thin film of Fig. 5 by a beam in normal incidence. It is well known that such a beam is composed by a sum of plane waves propagating in a given range of incidence angles around null incidence [8]. When the width of this range of incidence angles tends to zero, the width of the beam increases and tends to infinity, thus the beam becomes a plane wave.

In Fig. 6, three directions of propagation have been represented: the normal and the two edges of the range of the incidence angle. Due to the refractive phenomenon inside the material of small permittivity, the emerging beam should be much larger than the beam deduced from the laws of geometrical optics. However, this prediction must be corrected since the slice of homogeneous material acts like a Fabry–Perot interferometer. In other words, the different plane waves of the beam are reflected at air–dielectric interfaces, in such a way that the transmission factor depends on the angle of incidence. We can conjecture from this remark that when the thin film of Fig. 6 is illuminated by a Gaussian beam, the emerging beam could be very different from a Gaussian beam. If the transmission factor strongly depends on the angle of incidence, the emerging beam could be composed of some separate beams, like the separate rings generated by a Fabry–Perot.

4. Numerical results

4.1. Shift of the beam transmitted by a slice of photonic crystal

The first problem that arises is to determine the wavelength for which the permittivity of the photonic crystal is close to zero. With this aim, it can be observed that the phenomenon of beam shift schematised in Fig. 5 looks very similar to the Goos–Hanschen phenomenon [8], but for the transmitted wave. Remembering that the photonic origin of this phenomenon lies on a rapid variation of the argument of the reflected wave with incidence angle, we are led to the search for a rapid variation of the phase shift of the transmission factor of the photonic crystal with incidence. Studies of grating anomalies [12] have shown that rapid variation of the characteristics of the scattered wave with incidence angle and with wavelength happens together. One can find such a variation in Fig. 2, at the left-hand side of the gap, for example, at $\lambda = 2.5447$. This rapid variation of the phase shift corresponds to a sharp peak of transmission. Fig. 7 shows the transmittance and phase shift versus the angle of incidence at this wavelength. It is to be noticed that the phase variation between 0° and 8° is close to 200°. The same phenomenon arises at $\lambda = 2.5575$, while for other wavelengths the phase variation in the same range of incidence angles remains of the order of 10°. The two peaks observed in Fig. 7 are similar to the peaks generated by a Fabry–Perot interferometer when the incidence angle is varied, the wavelength being fixed.

Using the rectangular coordinate system depicted in Fig. 1, let the incident beam be limited in the x-direction. The only component of the s-polarized incident complex electric field can thus be expressed as the integral:

$$E^{\mathrm{inc}}(x,y) = \int_{-\infty}^{+\infty} A(\alpha) \exp(i\alpha x - i\beta(\alpha) y) \mathrm{d}\alpha, \qquad (1)$$

with $\alpha = k \sin \theta$, $\beta(\alpha) = \sqrt{k^2 - \alpha^2}$ and $k = 2\pi/\lambda$.

We consider Gaussian beams with mean incidence θ_0:

$$A(\alpha) = \frac{W}{2\sqrt{\pi}} \exp\left(- \frac{(\alpha - \alpha_0)^2 W^2}{4} \right), \qquad (2)$$

where $\alpha_0 = k \sin \theta_0$ and define the angular range as the range where the amplitude $A(\alpha)$ is greater than $A(\alpha_0)/10$. It can be noticed that the parameter W appearing in Eq. (2) is directly linked to the incident beam width.

Fig. 8 shows the field map (modulus of the electric field) around and inside the crystal, the incident field being

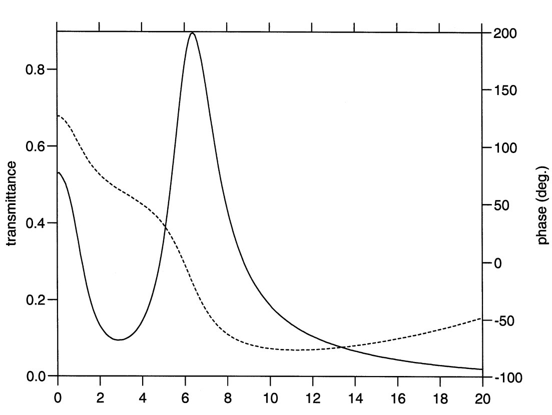

Fig. 7. Transmittance and phase shift at $\lambda = 2.5447$.

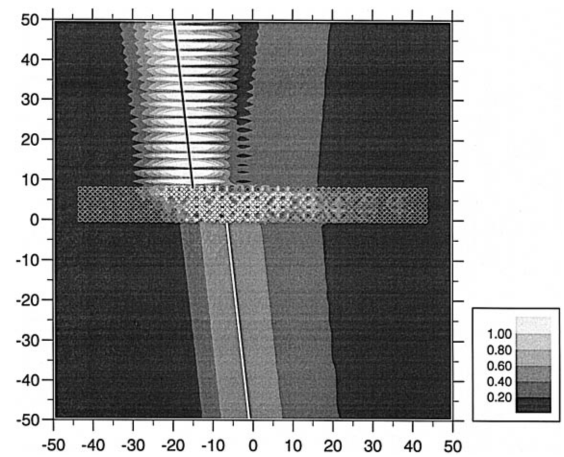

Fig. 8. Field map of the total field modulus with a photonic crystal similar to that of Fig. 2, but of finite extension in the x-direction. This crystal is made of 69×7 rods, as shown in the figure. Above the crystal the beam reflected by the crystal interferes with the incident beam and generates a system of stationary waves. Below the crystal, the figure shows the transmitted beam. Straight lines show the locus of the maximum incident (black line) and transmitted (white line) fields.

a Gaussian beam of wavelength $\lambda = 2.5447$, of angular range [1.7°, 11.2°], the mean incidence angle being equal to 6.4°. The calculation has been achieved using the rigorous theory of scattering by a finite number of rods [6]. The beam directly reflected by the crystal interferes with the incident beam and generates a system of stationary waves which can be observed at the top left corner of the figure. Below the crystal, the center of the transmitted beam is shifted to the right by four wavelengths with respect to the center of the incident beam, which shows that the permittivity of the effective medium is small. At the top right side of the figure, one can observe a second reflected beam

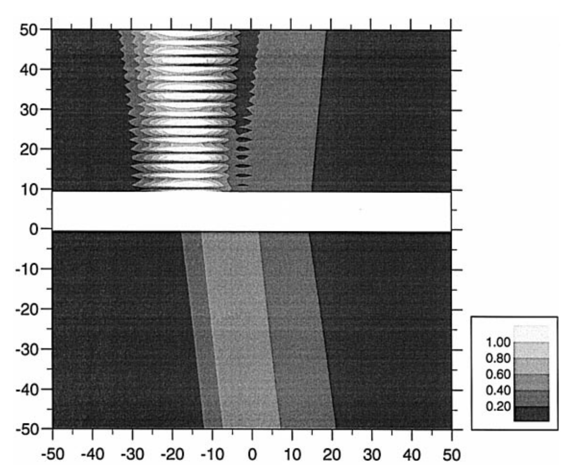

Fig. 9. The same as Fig. 8, but from another rigorous theory, the crystal of Fig. 8 having an infinite extension in the horizontal direction.

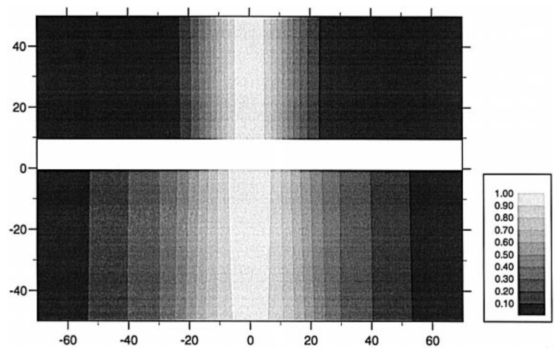

Fig. 10. Field map of the incident (above the photonic crystal) and transmitted fields (below the photonic crystal) modulus when the photonic crystal used in Fig. 9 is illuminated by a Gaussian beam in normal incidence. The transmitted field has been multiplied by a factor 2.25 in order to get the same maximum value as the incident field.

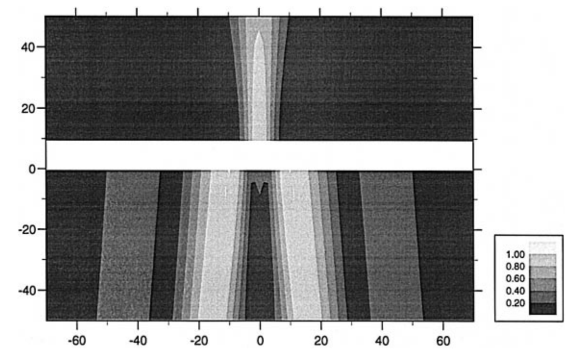

Fig. 12. Split of an incident beam (above the photonic crystal) into four separated transmitted beams (below the photonic crystal). The transmitted field has been multiplied by a factor 3.72.

generated by the light transmitted inside the crystal and then reflected. In order to check the validity of this surprising result, we achieved the same calculation using a quite different rigorous theory [7]. This theory is able to deal with periodic (in x) structures and thus the photonic crystal shown in Fig. 8 has been continued in the x-direction in order to become infinite. Fig. 9 shows the field above and below the crystal. The very good agreement between Figs. 8 and 9 shows that the edges of the crystal of Fig. 8 (in the x-direction) have no effect on the scattering phenomenon, a fact which could be predicted by observing in Fig. 8 that the field at the limits (in x) of the crystal vanishes. Furthermore, this agreement shows the validity of both calculations.

4.2. Widening and splitting of the transmitted beam

In order to check the predictions of Fig. 6, we have used the photonic crystal of Section 4.1 at the same wavelength but with a Gaussian beam in normal incidence. The angular width of the beam as previously defined is equal to 9.4°. Fig. 10 shows the field map of the incident beam (top) and the transmitted beam (bottom) obtained from the rigorous theory described in Ref. [4]. Obviously, the beam is significantly widened, even though this phenomenon is partially hidden by the fact that the transmitted beam contains 72% of the energy of the incident beam only. The structure of the field obtained by adjusting the maximum of the transmitted field modulus to the same value (unity) as the incident field modulus provides a better estimate of the widening, which is close to 200%.

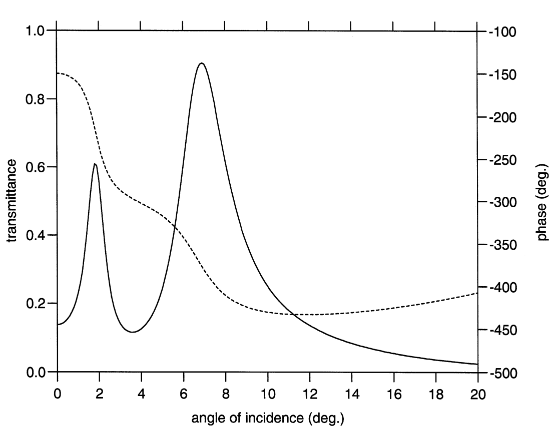

Fig. 11. The same as Fig. 7 but for $\lambda = 2.543$.

Looking at Fig. 7, it appears that the transmission factor contains three peaks, including negative incidence angles. It can be conjectured that the transmitted field could be split into three beams, the first is propagating along the y-axis and the two others are propagating symmetrically at an angle of $\pm 6.4°$ from the first one. In fact, the field map shows a complicated interference system between these three beams since they are not separated. On the other hand, a slight change of the wavelength from $\lambda = 2.5447$ to $\lambda = 2.543$, which significantly modifies the transmittance of the crystal (Fig. 11), leads to the transmitted field shown in Fig. 12, with four separated transmitted beams corresponding to the four peaks of the transmittance. The angular width of the incident beam extends from $-14.2°$ to $14.2°$. It is worth noticing that the four transmitted beams, which correspond to the rings of a Fabry–Perot, have very close directions of propagation despite the moderate width of the crystal. This is a consequence of the small value of the permittivity.

5. Conclusion

It has been proved from electromagnetic theory that ultrarefractive optics phenomena can be generated by dielectric photonic crystals.

Surprising phenomena like anomalous refraction, widening or splitting of a light beam have been shown.

In the present work, we have not tried to optimize the amplitude of ultrarefractive effects and thus it can be conjectured that more pronounced phenomena could be obtained if practical applications are envisaged.

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